due to the viscous flow strength of the fluid in the crack. This feature fundamentally distinguishes porous material rupture by a viscous fluid from homogeneous material rupture (for example, PMMA [4]).

It is clear from Figs. 5 and 6 that during rupture of a porous medium, just as for fracture of a nonporous one [4], there is a delay of the fluid front in the crack from its vertex ("spout"). However, the delay is more significant in the porous medium and grows with decreasing strength properties of the medium.

In conclusion, we note that a reduction of the viscosity of the working fluid, an increase in the percolation permeability of the porous medium and its strength leads to a decrease in the rupture zone dimensions. This is due to the growth of percolation loss and the quasi-brittle tensile strength of the medium.

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DO BRITTLE AND PLASTIC MATERIALS DIFFER WHEN SPALLING?

A. G. Ivanov and V. A. Ogorodnikov

UDC 620.178.7

We examine the energy description of spalling for brittle and plastic materials. We cite experimental data which justifies the use of one and the same relations for these materials.

Advances in fracture mechanics have demonstrated the fruitfulness of the energy approach in the description of brittle fracture. However, direct use of fracture mechanics is made difficult by the peculiarities of material fracture during spalling. Therefore, in [1], an attempt was made to use the balance of the elastic strain energy and the work of brittle fracture of the material as the necessary condition for failure, without imposing any sort of limitation on the failure mechanism itself.

This necessary condition can be written in the form

$$\int_0^{\delta} \frac{\sigma^2\, dx}{AE} \geqslant \lambda, \tag{1}$$

where σ is the tensile stress; x is the coordinate, reckoned from the free surface of the material; E is Young's modulus; λ the specific work of brittle material fracture per unit area; A is a function of Poisson's ratio ν, equal to $2(1-\nu)[(1+\nu)(1-2\nu)]^{-1}$.

It follows from (1) that the fracture stress σ_f and the thickness of the spallation layer of material δ are related by the inequality

$$\sigma_f^2 \delta \geqslant \alpha \lambda E A. \tag{2}$$

Arzamas. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 1, pp. 102-106, January-February, 1992. Original article submitted July 23, 1990.

0021-8944/92/3301-0091$12.50

Here, the coefficient α lies in the range from 1 to 3, and depends on the form of the incident pressure pulse. Thus, a triangular pulse corresponds to $\alpha = 3$, and a square pulse to $\alpha = 1$. We assume that (1) is also sufficient, since during spall, fracture is simultaneously initiated at a large number of points.*

In [1], by using $dp/dt = \sigma_f/2\delta = E\dot{\varepsilon}/2$, (1) is written for a triangular pulse in the form

$$\sigma_f = (6\lambda c\rho dp/dt)^{1/3} = (3\rho cE\lambda\dot{\varepsilon})^{1/3}, \tag{3}$$

where dp/dt is the pressure decay rate behind the incident wave front; $\dot{\varepsilon}$ is the strain rate in the tensile wave; ρ and c are the density and sound speed in an unbounded medium.

Somewhat later, a similar approach for describing material fragmentation during bulk dilatation, and also during spall was used in [3]. The corresponding equation for description of spall in brittle materials was obtained in the form

$$\sigma_f = \left(3\rho cK_m^2\dot{\varepsilon}\right)^{1/3}. \tag{4}$$

Since $K_m^2 = 2\gamma E$, and the magnitude of surface formation energy 2γ is analogous to λ, formulas (3) and (4) coincide.†

Being convinced that the descriptions of brittle materials in [1, 3] are identical, we turn to the description of plastic materials during spall. The concept of a brittle or plastic material is connected with material behavior during fracture. Without losing generality, for simplicity we limit consideration to materials with a bilinear stress–strain relation of the form $\sigma = \varepsilon E$ for $\varepsilon \leqslant \varepsilon_e$ and $\sigma = \sigma_e + (\varepsilon - \varepsilon_e)M$ for $\varepsilon \geqslant \varepsilon_e$ (σ_e and ε_e are the maximum values of σ and ε in the elastic region, and M is the strength modulus).

In a macroscopic treatment (without analysis of the fracture surface), the material is usually assumed to be brittle, if in the standard static tensile test conditions, it fractures for $\varepsilon_f \leqslant \varepsilon_e$, and plastic if $\varepsilon_f > \varepsilon_e$.

Note that this division is conditional. It is well known that material becomes more brittle with increasing test sample size, and with diminishing size, a brittle material can acquire plastic properties and fracture for $\sigma_f < \sigma_e$. Thus, the property of material plasticity or brittleness depends not only on temperature, strain rate and stress state, but also depends essentially on the size of the test sample [4].

During static elongation of samples of brittle and plastic materials, there is a significant difference in their failure. The failure of a brittle material takes place at the expense of elastic strain energy by passage of rapidly propagating cracks virtually in conditions of one-dimensional strain for $\sigma_f < \sigma_e$. The elastic strain energy is insufficient for failure of a plastic material. Before failure of a plastic material sample, the energy expended in plastic deformation of the entire sample is, as a rule, much greater than the energy spent directly on material division. The fracture process takes place in a uniaxial state. Does there remain such a significant difference between brittle and plastic materials undergoing shock-wave fracture by spallation? We say, no. Such an assertion is based on two reasons connected with shock-wave loading: the sharp increase in elastic strain energy and the small amount of energy being dissipated in plastic flow of the material in comparison with the elastic strain energy. Let us examine these reasons.

1. In view of the one-dimensional deformation of the material, the effective yield point grows by a factor of $k_1 = (1 - \nu)(1 - 2\nu)^{-1}$. For $\nu = 0.3$, $k_1 = 1.75$. As a consequence of the dynamic nature of loading, σ_e increases with growth in $\dot{\varepsilon}$ up to $\sim 10^4$-10^5 sec^{-1} for many materials (for soft steels, up to $k_2 = 4.5$ times). Thus the effective value of σ_e and the elastic strain energy tend to grow sufficiently rapidly to transfer the material from plastic to the brittle classification in the above-considered sense. So, the effective value of σ_e for soft steel increases by $k = k_1k_2 = 8$ times, and the elastic strain energy (assuming

*In the limiting case of the interaction of rarefaction waves, the characteristic separation between initiation points can be estimated as $\sim 10^{-6}$ m [2].

†The numerical agreement of the coefficients in (3) and (4) is evidently a coincidence. In [3], the ratio of the fracture surface area per unit volume was taken as $6/\delta$. For the one-dimensional case characterizing spall, this ratio is too small by a factor of three. In addition, the growth of stress with coordinates in the tensile wave was not taken into account. Therefore a difference in coefficients of a factor of 2-3 would be acceptable.

unchanging E) by a factor of 64. The threshold brittleness* of this material decreases from 640 to 10 mm. Thus during spall failure, soft steel must behave as a brittle material.

2. In the strong shock wave interval, for which spallation is possible ($\sigma > \sigma_e$), the energy dissipation in plastic flow is low. Two facts serve as evidence for this assertion: the conservation law of the doubling of the bulk material velocity during normal shock wave reflection from a free surface, and experiments on direct measurement of the hysteresis loop during shock loading and subsequent relief. So, for a typical representative of plastic materials (copper), the divergence of the shock and cold compression curves in the pressure-compression plane and as a consequence, violation of the doubling law, must be expected for $p > 40$ GPa. This is significantly higher than the spall stress interval. A similar picture characterizes soft aluminum and other plastic materials.

Information on the magnitude of the hysteresis loop during shock compression and subsequent relief, for example, for polycarbonate, was given in [5]. For compression up to $p \sim 0.5$ GPa, which is a few times larger than the spall stress, this value does not exceed 4%. A similar estimate for aluminum can be obtained from [6]. And here the hysteresis loop is several times smaller than the elastic strain energy.

These considerations give a basis for a descritpion of spall fragmentation of plastic materials (under normal conditions) similar to that considered for brittle materials. For both plastic and brittle materials, one can expect manifestation of scale effects of an energy nature, that is, relations (1)-(4) will be valid.

The approach adopted here to distinguish brittle from plastic materials is not unique. For their criteria, the form of the microdefects, the development, growth, and coalescense of which leads to formation of fracture surfaces in the material during spall were used in [3]. In [3], it is assumed that if fragmentation takes place through development of microcracks, then the material is brittle; if through the inception, growth, and coalescense of small cavities, it is plastic. The latter pertains to soft aluminum, copper, tin, and lead [3]. Such a definition of plastic materials gives a basis for using the quantity $\sigma_e\varepsilon_c = \delta$ as the specific surface energy in the energy balance equation ($\varepsilon_c \sim 0.15$ is the critical strain). The resultant fracture stress takes the form

$$\sigma_f = 2\rho c^2\sigma_e\varepsilon_c = 0{,}3\sigma_e E, \tag{5}$$

where σ_f does not depend on δ or $\dot{\varepsilon}$, since according to [3], the value of σ_e (as for E in [1, 3]) is taken to be constant.

Thus, according to [1], for geometrically similar changes of the dimensions of the experimental assembly, or for variation of $\dot{\varepsilon}$ for brittle and plastic materials, the following is valid:

$$\sigma_f^2\delta/\lambda = \text{const} \quad \text{or} \quad \sigma_f^3/(\lambda\dot{\varepsilon}) = \text{const}, \tag{6}$$

and according to [3], (6) also holds for brittle materials while at the same time (5) is valid for plastic materials, or

$$\sigma_f = \text{const}. \tag{7}$$

Let us turn to the experiment. In [3], soft aluminum and copper were treated as plastic materials. Is condition (7) satisfied by these materials? No. A series of works have used experimental results [7] on the spall strength of soft aluminum. From these, it follows that for a seven-fold increase in $\dot{\varepsilon}$, σ_f grows by a factor of 2.8. Data for copper and lead were also given. For copper, a 3.2-fold increase in $\dot{\varepsilon}$ results in an increase in σ_f by a factor of 1.8. The tendency towards growth in σ_f with increasing $\dot{\varepsilon}$ is also observed for lead, as noted in [8] as well.

The spall strength of copper with change in $\dot{\varepsilon}$ over a broad range (by a factor of $10\text{-}10^3$) was also studied in [9-11]. In $\sigma_f - \delta$ coordinates, these results can be described by $\sigma_f^n\delta = \text{const}$. For the experimental data in [9], $n \simeq 6.4$, while for [10, 11], $n \simeq 5.3$ and 5.4, respectively. Thus there are experimental results for the plastic materials used in the analysis in [3] which do not support (7).

In our opinion, the reason (7) is not satisfied is connected with the use of λ, represented by $\sigma_e\varepsilon_c\delta$ as the specific surface energy for plastic materials. Such a representation

*According to the definition in [4], the threshold brittleness L_0 is the minimum value for a cubic rib being extended on opposite sides with a force σL^2. In this case, the elastic strain energy is still sufficient for brittle fracture ($L_0 = 2\lambda E/\sigma_e^2$).

is equivalent to the assumption that structural defects accumulate throughout the entire material volume. As numerous experiments have shown, the defects are localized in the neighborhood of the fracture surface, regardless of the micromechanism of fracture development (microcracks or small cavities). Therefore, there is no basis for a change of the right-hand side of the energy balance equation (1) with change in the fracture micromechanism. And this signifies that there is no fundamental difference between brittle and plastic materials during spall.

The first attempts to experimentally verify (6) for spall, with the assumption λ = const were made in [12]. In that work, it was noted that the actual values for the power exponent for σ_f in (6) varies from 2 to 5.2 for different materials (steel, aluminum alloys, copper, titanium, aluminum, organic glass). The average value is ~3.5. The spallation of an aluminum alloy (σ_e = 333 MPa, σ_f = 449 MPa) was studied in [13]. The experiments were done using impacting plates which had geometrically similar assemblies with a four-fold change in scale. The results obtained are well represented by $\sigma^n t$ = const for n = 2.63. The same relation with $n \approx 3.4$ also describes experiments with uranium [3]. Thus the dependence of σ_f on δ or t is weaker than (6) [t is the duration time of the tensile pulse]. The most significant reason is the dependence $\lambda(t)$.* The assumption that λ = const is, strictly speaking, indefensible. Thus, it is known that even for a material as brittle as glass, 2γ (the analog of λ) is significantly greater than the specific energy of free surface formation λ_0, and that this difference can be up to 10^4 times larger for different materials. Where is this energy expended during spallation? Fracture with energy expenditure λ_0 can be attained by applying the tensile forces to two neighboring crystal planes. During real fracture, the forces on these planes are conveyed through a number of intermediate planes. The latter in no way differs from what is being considered. It is natural to assume that there will be partial fracture in these as well, which is confirmed in microphotographs.

Thus with decreasing tensile pulse duration, down to $t \sim 10^{-13}$ sec, the value of λ must drop to λ_0, while σ_f grows to the theoretical strength σ_{f0}. According to [12], for copper, $\lambda \approx 4$ J/cm^2 and σ_f = 0.75 GPa when $t = 3 \cdot 10^{-7}$ sec; while $\lambda_0 = 3.3 \cdot 10^{-4}$ J/cm^2 and σ_{f0} = 35 GPa when $t \sim 10^{-13}$ sec. Assuming $\sigma^n t$ = const, by using σ_f and t and setting λ = const, we obtain $n \approx 3.9$. By using the other pair of independent values of λ and t and setting $\lambda \sim t^m \sim \delta^m$, we find m = 0.63. Substituting $\lambda(t)$ into (6), we have $\sigma^{5,4} t$ = const, or $\sigma^{5,4}\delta$ = const. These examples of computing $\lambda(t)$ for spall fracture indicate that the real dependence $\sigma_f(\delta)$ is considerably weaker and the value of the exponent in (1), (2), etc. may be 1.5-3 times larger than 2. This result makes it possible to reduce the acuteness of the problem of the discrepancy in the experimentally determined exponents lying between 2 and 5.2 for a number of metals [13]. We should note that [15] was the first work to draw attention to the possible dependence $\lambda(t)$.

Thus, regardless of the form of material fracture during static elongation (brittle or plastic), or the mechanism of fracture formation and development at the microlevel (by means of microcracks or of small cavities), there is no fundamental difference in the energy description of spall failure of materials: in view of the characteristics of spall fracture, the most conservative value of λ depends on the duration time of the tensile forces (or on the scale of the object). Therefore, the real value of the exponent on σ_f in (2) and its analogs can grow by a factor of 1.5-3.

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*In estimating $\lambda(t)$, [14] erroneously indicated that this dependence was weak.

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THE MOST RESTRICTIVE BOUNDS ON CHANGE IN THE APPLIED ELASTIC CONSTANTS FOR ANISOTROPIC MATERIALS

N. I. Ostrosablin

UDC 539.3:548.053

A representation of the elastic constants tensor was given in [1], for the general anisotropic case, in a form which ensured positive definiteness of the specific strain energy and which indicates the strictest bounds for each elastic constant. In the same paper, there is reference to works in which this question had been previously addressed, and which studied the general properties of the tensor of elastic constants. The limits of the variability of the elastic constants was also studied in [2, 3]. Formulas for the characteristic elastic moduli and the states for materials of all crystallographic systems were obtained in [4].

In this work, explicit formulas for the applied elastic constants (Young's modulus, the shear and bulk moduli, Poisson's ratio) are given on the basis of the representation from [1] for the general anisotropic case. The formulas show the limits of variability of these constants. The appropriate formulas for the elastic constants for materials of all crystallographic systems are given. The strictest bounds (without refinement) on these constants which ensure a positive definite specific strain energy are established.

1. In the matrix notation of [1, 4], Hooke's law and the specific strain energy are written as

$$\sigma_i = A_{ij}\varepsilon_j, \quad \varepsilon_i = a_{ij}\sigma_j; \tag{1.1}$$

$$2\Phi = \sigma_i\varepsilon_i = A_{ij}\varepsilon_i\varepsilon_j = a_{ij}\sigma_i\sigma_j. \tag{1.2}$$

Here and below, repeated indices denote summation from 1 to 6. The matrices of the elastic constants A_{ij} and a_{ij} are symmetric, and the quadratic form (1.2) is positive definite.

As shown in [1], a_{ij} and A_{ij} can be represented in the form

$$a_{ij} = d_1c_{i1}c_{j1} + d_2c_{i2}c_{j2} + d_3c_{i3}c_{j3} + d_4c_{i4}c_{j4} + d_5c_{i5}c_{j5} + d_6c_{i6}c_{j6}, \tag{1.3}$$
$$c_{ip} = 0 \ (p > i), \ c_{11} = \ldots = c_{66} = 1;$$

$$A_{ij} = d_1^{-1}c_{1i}^{-1}c_{1j}^{-1} + d_2^{-1}c_{2i}^{-1}c_{2j}^{-1} + d_3^{-1}c_{3i}^{-1}c_{3j}^{-1} + d_4^{-1}c_{4i}^{-1}c_{4j}^{-1} + d_5^{-1}c_{5i}^{-1}c_{5j}^{-1} + d_6^{-1}c_{6i}^{-1}c_{6j}^{-1}, \tag{1.4}$$
$$c_{ip}^{-1} = 0 \quad (p > i), \quad c_{11}^{-1} = \ldots = c_{66}^{-1} = 1.$$

Novosibirsk. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 1, pp. 107-114, January-February, 1992. Original article submitted July 12, 1990; revision submitted November 23, 1990.

0021-8944/92/3301-0095$12.50